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FUNDAMENTAL REGULARITIES IN THE MOTION OF AN ARBITRARY GAS

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The problem of the derivation of the equation of state for real, dissociating, and ionizing gases expressed in terms of the similarity numbers for gaseous flows and the determination of the most general connections among heterogeneous variables are given.

We write the equation of state in the most general form

$$pu = Z_{\text{ef}} \frac{R}{\mu} T, \quad Z_{\text{ef}} = Z\varepsilon. \tag{1}$$

Here Z designates the coefficient of compressibility, and ε is the reduced number of moles obtained as a result of dissociation. For example, for dissociating nitrogen tetroxide

$$\varepsilon = 1 + \alpha_{10} + \alpha_{10}\alpha_{20},$$

where α_{10} and α_{20} are the degrees of dissociation of the first and second stages of the reaction in the reactive system $N_2O_4 \rightleftarrows 2NO_2 \rightleftarrows 2NO + O_2$. Below, we will operate with the effective coefficient (Z_{ef}), which takes account of the dissociation of molecules as well as the nonideality of gas.

When considering a real gas in the absence of dissociation $\varepsilon = 1$ and $Z_{\text{ef}} = Z$, i.e., everywhere when applied to a real gas, we will treat Z_{ef} as a coefficient of compressibility.

If we consider, as an arbitrary gas, a diatomic gas with account for dissociation and ionization, then

$$Z_{\text{ef}} = 1 + \alpha + 2\alpha_e,$$

where α is the degree of dissociation and α_e is the degree of ionization.

The velocity of sound in an arbitrary gas can be represented as

$$a = y a_{\text{id}}, \tag{2}$$

where y is a correcting coefficient, and a_{id} is the velocity of sound in an ideal gas:

$$a_{\text{id}} = \sqrt{k \frac{R}{\mu} T}. \tag{3}$$

In order to derive an analytic expression for the correcting coefficient y, we perform the following transformations. We use the differential equations of thermodynamics in the form

$$C_p - C_v = -T \frac{\left[\left(\frac{\partial v}{\partial T}\right)_p\right]^2}{\left(\frac{\partial v}{\partial p}\right)_T}, \tag{4}$$

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$$\left(\frac{\partial p}{\partial v}\right)_s = \frac{C_p}{C_v}\left(\frac{\partial p}{\partial v}\right)_T . \tag{5}$$

As is known, the square of the velocity of sound in an arbitrary gas can be written as

$$a^2 = -v^2\left(\frac{\partial p}{\partial v}\right)_s . \tag{6}$$

Dividing both parts of (4) by C_p, solving simultaneously Eqs. (1), (5), and (6), and considering C_p in the general case as the effective isobaric heat capacity (if we consider a dissociating gas), we obtain

$$a^2 = -\left\{\left(\frac{\partial v}{\partial p}\right)_T + \frac{T}{C_{p\,\mathrm{ef}}}\left[\left(\frac{\partial v}{\partial T}\right)_p\right]^2\right\}^{-1}\left(Z_{\mathrm{ef}}\frac{R}{\mu}\frac{T}{p}\right)^2 . \tag{7}$$

From (1) we obtain the following partial differential equations, which enter into (7),

$$\left(\frac{\partial v}{\partial p}\right)_T = -\frac{RT}{\mu p^2}\left[Z_{\mathrm{ef}} - p\left(\frac{\partial Z_{\mathrm{ef}}}{\partial p}\right)_T\right], \tag{8}$$

$$\left(\frac{\partial v}{\partial T}\right)_p = \frac{R}{\mu p}\left[Z_{\mathrm{ef}} + T\left(\frac{\partial Z_{\mathrm{ef}}}{\partial T}\right)_p\right]. \tag{9}$$

Equation (7), taking account of (8) and (9) and of the notation

$$\eta = Z_{\mathrm{ef}} - p\left(\frac{\partial Z_{\mathrm{ef}}}{\partial p}\right)_T , \tag{10}$$

$$\omega = Z_{\mathrm{ef}} + T\left(\frac{\partial Z_{\mathrm{ef}}}{\partial T}\right)_p \tag{11}$$

can be written in the form

$$a = \sqrt{k\frac{RT}{\mu}}\;\frac{Z_{\mathrm{ef}}}{\sqrt{k\left(\eta - \frac{R}{\mu C_{p\mathrm{ef}}}\,\omega^2\right)}} . \tag{12}$$

Equation (12) represents the velocity of sound in an arbitrary gas, where the correcting coefficient is in correspondence with (2)

$$y = \frac{Z_{\mathrm{ef}}}{\sqrt{k\left(\eta - \frac{R}{\mu C_{p\mathrm{ef}}}\,\omega^2\right)}} . \tag{13}$$

In [1], the following expanded expressions for the coefficients η and ω are obtained for the dissociating nitrogen tetroxide with account for partial derivatives in (10) and (11).

The details of the effect of the coefficient Z_{ef} on different thermogasdynamic dependences are presented in [2].

If the gas under consideration is nondissociating but real, then instead of Z_{ef} the coefficient of compressibility will be Z. In this case partial derivatives can be determined by means of graphical differentiation from experimental diagrams of compressibility.

In accordance with (2) and (3) the expressions for the critical velocity can be written in the form

$$a_{\mathrm{cr}} = y_{\mathrm{cr}}\sqrt{k\frac{R}{\mu}T_{\mathrm{cr}}} . \tag{14}$$

We use the concepts given in [3]:

$$\tau = \frac{T}{T_{\mathrm{cr}}} , \tag{15}$$

$$\pi = \frac{p}{\rho w a_{\text{cr}}}. \tag{16}$$

Substituting the value of p/ρ in (16) from (1) and taking account of (14), (15), we obtain

$$\frac{\pi\lambda}{\tau}\frac{y_{\text{cr}}^2}{Z_{\text{ef}}} = \frac{1}{k}. \tag{17}$$

Equation (17) contains important information. Being free of all constraints (typical of the ideal gas condition), Eq. (17) turns out to be applicable in the most general situations due to the introduction of two multipliers, the first of which is a correcting coefficient for the equation of state, and the second, for the square of the velocity of sound as a function of temperature in the critical state.

From Eq. (16) we can see that the reduced pressure depends on the static pressure p and the density ρ. The value τ, entering into (17), is related to the thermodynamic temperature ($\tau = T/T_{\text{cr}}$), i.e., we have in (17) the thermal quantities p, ρ, and T. Besides, (17) contains λ, which is one of the similarity numbers for gaseous flows ($\lambda = w/a_{\text{cr}}$), where the role of the constant is played by a physical constant (1/k). Equation (17) plays the role of the equation of state under conditions of flow and appears useful in calculations and also in an analysis of the physical features of the process; therefore, expansion of its region of applicability is undoubtedly of important significance.

Coefficients Z_{cr} and y_{cr}, which have a clear physical meaning, represent fairly complicated theoretical expressions. In Z_{cr}, the nonideality of the gas, which is due to the effect of forces of attraction and repulsion of gas molecules, dissociation, and the effect of forces of attraction and repulsion on dissociation, is taken into account. However, this complexity is justified; it is dictated by the high complexity of the process itself.

In Eq. (17), an interesting fact is exhibited: in all the regimes of flow the complex $\frac{\pi\lambda}{\tau}\frac{y_{\text{cr}}^2}{Z_{\text{ef}}}$ remains constant and equal to 1/k. Thus, this complex is invariant, with numerical values equal to: for monoatomic gases (k = 1.67) 0.585, for diatomic gases (k = 1.4) 0.714, and for triatomic and polyatomic gases (k = 1.3) 0.77.

We also note that in the given complex all three reduced parameters and the two correcting multipliers are combined.

With the help of identical transformations we can obtain different modifications of Eq. (17).

Let us derive the same complex, but expressed in terms of the Mach number. We transform the multiplier $Z_{\text{ef}}/y_{\text{cr}}^2$. We take the values of Z_{ef} from (1) and y_{cr}^2 from (14); taking account of (15), we can write

$$\frac{Z_{\text{ef}}}{y_{\text{cr}}^2} = \frac{p}{\rho}\frac{k}{\tau}\frac{1}{a_{\text{cr}}^2}. \tag{18}$$

Substituting the value of p/ρ from (1) into (18) and taking account of (3), we have

$$\frac{1}{y_{\text{cr}}^2} = \frac{1}{\tau}\left(\frac{a_{\text{id}}}{a_{\text{cr}}}\right)^2. \tag{19}$$

The equation of state for an ideal gas is

$$pv_{\text{id}} = \frac{R}{\mu}T. \tag{20}$$

Comparing (20) and (1), we obtain

$$u = Z_{\text{ef}}v_{\text{id}}. \tag{21}$$

The continuity equation for an ideal gas is

$$Gu_{\text{id}} = Fw_{\text{id}}. \tag{22}$$

The continuity equation for a real or dissociating gas is

$$Gv = Fw. \tag{23}$$

Solving simultaneously (22) and (23) with consideration of (21), we have

$$w = Z_{ef} w_{id}. \tag{24}$$

For an ideal gas

$$M_{id} = \frac{w_{id}}{a_{id}}. \tag{25}$$

For a real or dissociating gas

$$\lambda = \frac{w}{a_{cr}} = \frac{Z_{ef} w_{id}}{a_{cr}}. \tag{26}$$

Substituting the value w_{id} in (26) from (25), we write

$$\lambda = Z_{ef} M_{id} \frac{a_{id}}{a_{cr}}$$

or after simple transformations

$$\left(\frac{a_{id}}{a_{cr}}\right)^2 = \frac{1}{Z_{ef}^2} \frac{\lambda^2}{M_{id}^2}. \tag{27}$$

From (27), Eq. (19) can be represented in the form

$$\frac{1}{y_{cr}^2} = \frac{1}{Z_{ef}^2} \frac{1}{\tau} \frac{\lambda^2}{M_{id}^2}. \tag{28}$$

Substituting (28) in (17), we have

$$\frac{\pi Z_{ef} M_{id}^2}{\lambda} = \frac{1}{k}. \tag{29}$$

From (24) and (25) we obtain

$$M_{id}^2 = \frac{w^2}{Z_{ef}^2 a_{id}^2},$$

or taking account of (2)

$$M_{id}^2 = \frac{w^2 y^2}{Z_{ef}^2 a^2} = M^2 \frac{y^2}{Z_{ef}^2}. \tag{30}$$

Substituting the value M_{id}^2 from (30) into (29):

$$\frac{\pi M^2 y^2}{Z_{ef} \lambda} = \frac{1}{k}. \tag{31}$$

Equation (31) is a modification of Eq. (17). The left side is an invariant. Here both similarity numbers (M and λ) occur for gaseous flows. Equation (31) can be used in calculations when it is necessary to use the Mach number.

When performing calculations according to Eq. (31), the value of y is determined from (13) with subsequent use of (10) and (11) by the method described in [1, 2]. When performing calculations according to (15), however, the value of y^2 characterizing critical conditions is determined from the same dependences (13), (10), and (11), but for the critical conditions it is written in the form

$$y_{cr} = \frac{(Z_{ef})_{cr}}{\sqrt{k\left[\eta_{cr} - \frac{R}{\mu (C_{p\,ef})_{cr}} \omega_{cr}^2\right]}}, \tag{13'}$$

$$\eta_{cr} = (Z_{ef})_{cr} - p\left[\frac{(\partial Z_{ef})_{cr}}{\partial p}\right]_T, \tag{10'}$$

$$\omega_{cr} = (Z_{ef})_{cr} + T_{cr}\left[\frac{(\partial Z_{ef})_{cr}}{\partial T}\right]_{p_{cr}}. \tag{11'}$$

In conclusion, we would like to draw attention to the fact that Eqs. (17) and (31) are original because they combine thermal quantities and the flow velocity. On the other hand, the invariance makes it possible to solve further problems of applied thermogasdynamics and to investigate a complicated process of flow (dissociation of molecules, etc.). Studies of the obtained dependences disclose fundamental features of the motion of an arbitrary gas and promote the solution of problems which are of actual significance in contemporary technology involving high-speed flows of real gases.

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SOME FEATURES OF THE THERMAL STRATIFICATION OF LIQUIDS DURING NATURAL CONVECTION IN CYLINDRICAL CAVITIES WITH ANNULAR RIBS

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The effect of annular ribs on the temperature of the free surface of a liquid during natural convection in a heated vertical cylindrical cavity has been investigated. A method is proposed for calculating the limiting value of this temperature.

Free convective motions occur in a liquid during heat exchange between a container or cavity and the surroundings which lead to nonlinear temperature distributions over its volume, or thermal stratification. This thermal stratification can have a significant effect on the intensity of the thermophysical processes occurring in the container, as a result of which its study is of practical interest.

Real containers often contain framing elements or other devices which are positioned in the zone of the free-convection boundary layer and which have an effect on the temperature distribution over the volume of the liquid. An experimental investigation into the effect of ribs on thermal stratification was carried out in [1-3]. It was found that the presence of ribs leads to a decrease in the vertical temperature gradient of the liquid on average with respect to the height of the part of the container. The results of experiments with visualization of the liquid flows have been given. It was noted in [1] that the temperature of the free surface of the liquid in the container with ribs was somewhat higher than in the container without ribs, while the opposite result was found in [2, 3]. There are no methods available for calculating the thermal stratification in containers with ribs.

In the present paper an investigation has been made into the effect of annular ribs on the maximum temperature of a liquid in a heated cavity or container, which is the temperature of the free surface, and a method is proposed for calculating the maximum value of this temperature. For carrying out the experiments two cylindrical containers were used (Fig. 1) with heights of 0.25 and 0.75 m and diameters of 0.25 m. The cylindrical part of each vessel was made of stainless steel of thickness $2 \cdot 10^{-3}$ m and at the bottom end had a

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